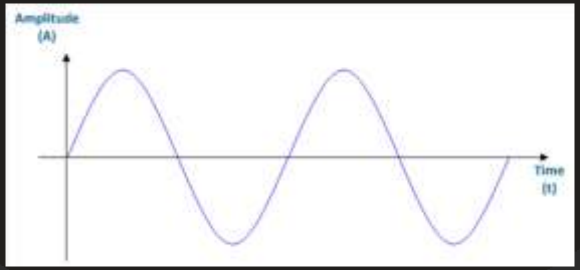
INTRODUCTION TO DIGITAL SIGNAL PROCESSING



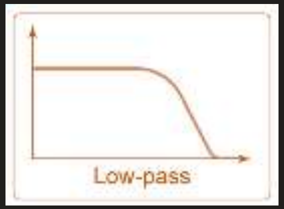
Analog

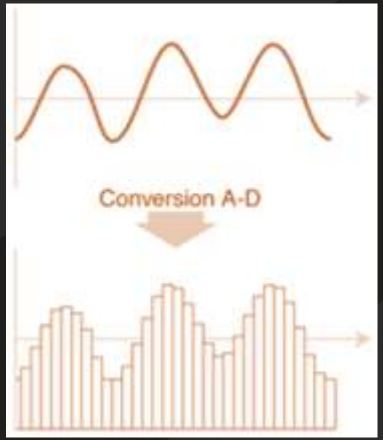
Digital

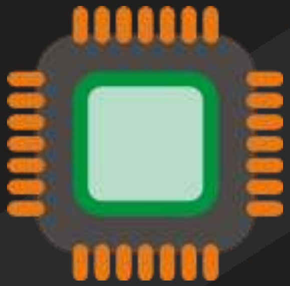
- Analysis, synthesis and modify
- Analog signal processing
- Digital signal processing

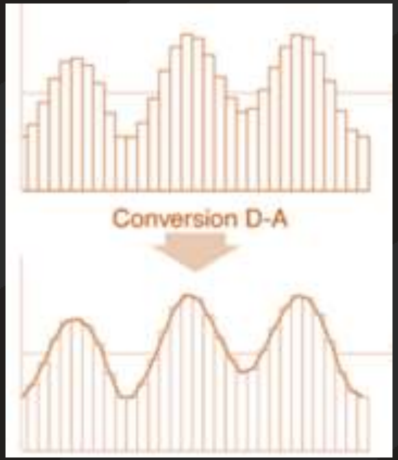
BASIC BLOCKS OF DIGITAL SIGNAL PROCESSING

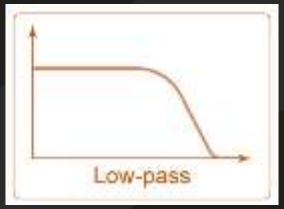
Analog input → Pre filter → AD Converter → Digital Signal Processor → DA Converter → Post filter → Analog output











Discrete Fourier Transform (DFT)

- DFT is a powerful computation tool which allows us to evaluate the Fourier transform on a digital computer or specifically designed hardware
- We notate like this

$$X(k) = DFT[x(n)] \qquad x(n) = IDFT[X(k)]$$

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}} \;, 0 \leq \mathrm{k} \leq \mathrm{N}-1$$

IDFT

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}} \;, 0 \leq \mathrm{n} \leq \mathrm{N}-1$$

Let us define a term $W_N = e^{-\frac{j2\pi}{N}}$ which is known as twiddle factor and substitute in above equations

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \;, 0 \leq \mathrm{k} \leq \mathrm{N}-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) w_N^{-nk} \;, 0 \leq \mathrm{n} \leq \mathrm{N}-1$$

Let us take an example

Q) Find the DFT of the sequence $x(n) = \{1,1,0,0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}} , 0 \le k \le N-1$$

$N = 4$

$$X(k) = \sum_{n=0}^{3} x(n)e^{-\frac{j2\pi kn}{4}} , 0 \le k \le N-1$$

$k = 0$

$$X(0) = \sum_{n=0}^{3} x(n)e^{-\frac{j2\pi 0n}{4}}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0 \quad = 2$$

$k = 1$

$$X(1) = \sum_{n=0}^{3} x(n)e^{-\frac{j\pi n}{2}}$$

$$= x(0) + x(1)e^{-\frac{j\pi}{2}} + x(2)e^{-j\pi} + x(3)e^{-\frac{j3\pi}{2}}$$

$$= 1 + 1\left[\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right] + 0 + 0 \quad = 1 - j$$

$k = 2$

$$X(2) = \sum_{n=0}^{3} x(n)e^{-\frac{j\pi 2n}{2}}$$

$$= x(0) + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1 + 1[\cos\pi - j\sin\pi] + 0 + 0 \quad = 1 - 1 = 0$$

$k = 3$

$$X(3) = \sum_{n=0}^{3} x(n)e^{-\frac{j\pi 3n}{2}}$$

$$= x(0) + x(1)e^{-\frac{j3\pi}{2}} + x(2)e^{-j3\pi} + x(3)e^{-\frac{j9\pi}{2}}$$

$$= 1 + 1\left[\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right] + 0 + 0 \quad = 1 + j$$

$$X(k) = \{2, 1 - j, 0, 1 + j\}$$

DFT as linear transformation (Matrix method)

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{W_N^{nk}} \text{ ,} 0 \leq \text{k} \leq \text{N} - 1$$

$$W_N = e^{-\frac{j2\pi}{N}}$$

Lets put $n = 0,1,2, \dots N\text{-}1$

$$X(k) = x(0).1 + x(1).W_N^{1k} + x(2).W_N^{2k} + \cdots + x(N-1)W_N^{(N-1)k}$$

$k = 0$

$$X(0) = x(0) + x(1) + x(2) + \cdots + x(N-1)$$

$k = 1$

$$X(1) = x(0) + x(1).W_N^1 + x(2).W_N^2 + \cdots + x(N-1)W_N^{(N-1)}$$

$\vdots$

$k = N-1$

$$X(N-1) = x(0) + x(1).W_N^{(N-1)} + x(2).W_N^{2(N-1)} + \cdots + x(N-1)W_N^{(N-1)(N-1)}$$

We can also represent the equation in matrix format

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ \vdots \\ X(N-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & \dots & W_N^{(N-1)} \\ 1 & W_N^2 & W_N^4 & W_N^6 & \dots & W_N^{2(N-1)} \\ 1 & W_N^3 & W_N^6 & W_N^9 & \dots & W_N^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & W_N^{(N-1)} & W_N^{2(N-1)} & W_N^{3(N-1)} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ \vdots \\ x(N-1) \end{bmatrix}$$

$$X_N = W_N x_N$$

$$x_N = W_N^{-1} X_N$$

$$W_N^{-1} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & W_N^{-1} & \ddots & W_N^{-(N-1)} \\ \vdots & \vdots & & \\ 1 & W_N^{-(N-1)} & \dots & W_N^{-(N-1)(N-1)} \end{bmatrix}$$

IDFT

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)w_N^{-nk} \text{ ,} 0 \leq \text{n} \leq \text{N} - 1$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left(w_N^{nk}\right)^*$$

Comparing we get

$$W_N^{-1} = \frac{1}{N} W_N^*$$

Symbolically we can write as

$$x(n) = \frac{1}{N} X_N W_N^*$$

Twiddle factor matrix

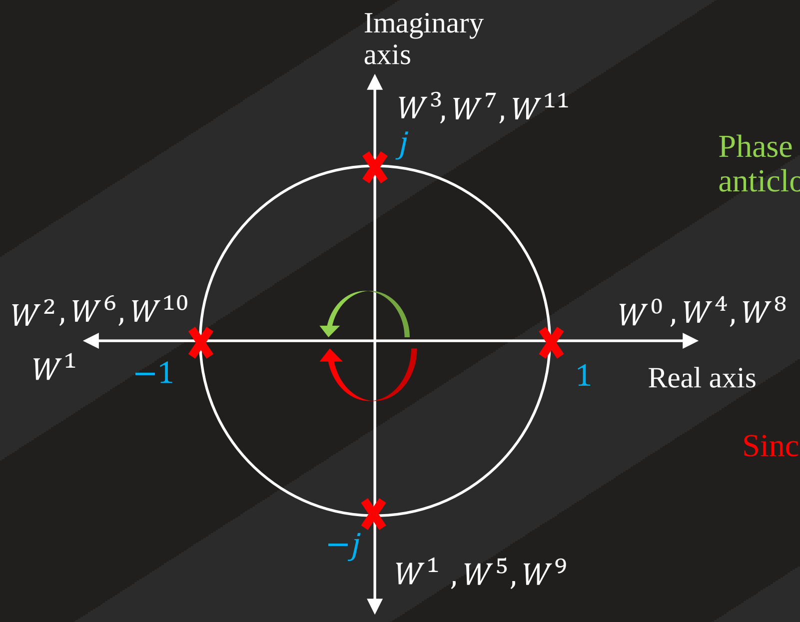
$$W_N = e^{-\frac{j2\pi}{N}}$$

Lies on the unit circle in the complex plane from 0 to 2π angle and it gets repeated for every cycle

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$W_2 = \begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$



Phase change (0^0 - 360^0) - anticlockwise

Since $e^{-j\theta}$ – clockwise

Relationship of the DFT to Fourier Transform

Fourier-Transform

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)e^{-j\omega n}$$

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}}$$

Comparing the above equations we get to find that DFT of x(n) is a sampled version of the FT of the sequence

$$X(k) = X(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{N}}, \qquad k = 0,1,2,\ldots N-1$$

Relationship between DFT & Fourie Transform

Relationship of the DFT to Z-Transform

Z-Transform

$$X(Z) = \sum_{n=0}^{N-1} x(n) z^{-n}$$

IDFT

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}$$

Substitute the value of x(n)

$$X(Z) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}\right] z^{-n}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[\sum_{n=0}^{N-1} e^{\frac{j2\pi kn}{N}} z^{-n}\right]$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[\sum_{n=0}^{N-1}\left(e^{\frac{j2\pi k}{N}} z^{-1}\right)^{n}\right]$$

$$\sum_{k=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$X(z) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[\frac{1-\left(e^{\frac{j2\pi k}{N}} z^{-1}\right)^{N}}{1-e^{\frac{j2\pi k}{N}} z^{-1}}\right]$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[\frac{1-e^{j2\pi k} z^{-N}}{1-e^{\frac{j2\pi k}{N}} z^{-1}}\right]$$

In the above condition $e^{j2\pi k} = 1$ for all the values of k

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[\frac{1-z^{-N}}{1-e^{\frac{j2\pi k}{N}} z^{-1}}\right]$$

$$X(z) = \frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\left[\frac{X(k)}{1-e^{\frac{j2\pi k}{N}} z^{-1}}\right]$$

Relationship between DFT & Z- Transform

Properties of Discrete Fourier Transform

Periodicity

If X(k) is N-point DFT of a finite duration sequences x(n) then

$$x(n+N) = x(n) \quad for\ all\ n$$
$$X(k+N) = X(k) \quad for\ all\ k$$

Linearity

If two finite sequences $x_1(n)$ and $x_2(n)$ are linearly combined as

$$x_3(n) = ax_1(n) + bx_2(n)$$

Then DFT of the sequence

$$X_3(k) = aX_1(k) + bX_2(k)$$

$$ax_1(n) + bx_2(n) \overset{DFT}{\Longleftrightarrow} aX_1(k) + bX_2(k)$$

Circular time shift

If X(k) is N-point DFT of a finite duration sequences x(n) then

$$DFT\left\{x((n-m))_N\right\} = X(k)e^{-\frac{j2\pi km}{N}}$$

Proof

IDFT

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi kn}{N}} \quad , 0 \leq n \leq N-1$$

Put n=n-m

$$x(n-m) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi k(n-m)}{N}}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi kn}{N}} e^{\frac{-j2\pi km}{N}}$$

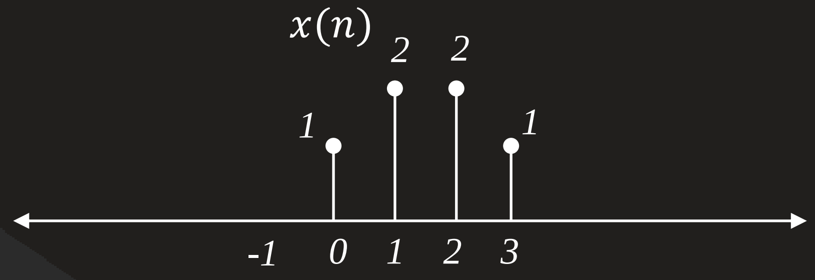
$$x(n-m) = x(n)e^{\frac{-j2\pi km}{N}}$$

Take DFT on both sides

$$DFT\{x(n-m)\} = X(k)e^{\frac{-j2\pi km}{N}}$$

Q) Consider a finite length sequences x(n) shown in figure. The five point DF of x(n) is denoted by X(k). Plot the sequences whose DFT is

$$Y(k) = e^{\frac{-4\pi k}{5}} X(k)$$



$$DFT\left\{x\left((n-m)\right)_N\right\} = X(k)e^{-\frac{j2\pi km}{N}}$$

Solution

$$DFT\left\{x\left((n-2)\right)_5\right\} = X(k)e^{-\frac{j2\pi k2}{5}}, n = 0,1,\ldots,4$$

For n=0 ➔ $y(0) = x\left((0-2)\right)_5 = x(5+0-2) = x(3) = 1$

For n=1 ➔ $y(1) = x\left((1-2)\right)_5 = x(5+1-2) = x(4) = 0$

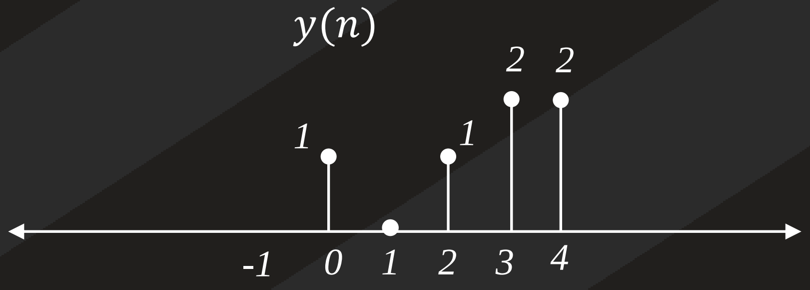
For n=2 ➔ $y(2) = x\left((2-2)\right)_5 = x(5+2-2) = x(5) = x(5-5) = x(0) = 1$

Exceeding the limit $0 \leq n \leq 4$

For n=3 ➔ $y(3) = x\left((3-2)\right)_5 = x(5+3-2) = x(6) = x(6-5) = x(1) = 2$

For n=4 ➔ $y(4) = x\left((4-2)\right)_5 = x(5+4-2) = x(7) = x(7-5) = x(2) = 2$

$$y(n) = \{1,0,1,2,2\}$$



Properties of Discrete Fourier Transform

Time reversal of the sequence

The time reversal of N-point sequence x(n) is attained by wrapping the sequence x(n) around the circle in clockwise direction.

$$x((-n))_N = DFT\{x(N-n)\} = X(N-k)$$

Circular frequency shift

If X(k) is N-point DFT of a finite duration sequences x(n) then

$$DFT\left[x(n)e^{\frac{j2\pi ln}{N}}\right] = X((k-l))_N$$

Proof

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}} \quad ,0 \le \mathrm{k} \le \mathrm{N}-1$$

Put k=k-l

$$X(k-l) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi(k-l)n}{N}}$$

$$= \sum_{n=0}^{N-1} x(n)ne^{-\frac{j2\pi kn}{N}} e^{\frac{j2\pi ln}{N}}$$

$$x(k-l) = X(k)e^{\frac{j2\pi ln}{N}}$$

Take DFT on both sides

$$X((k-l))_N = DFT\left\{x(n)e^{\frac{j2\pi ln}{N}}\right\}$$

Properties of Discrete Fourier Transform

Complex conjugate property

If X(k) is N-point DFT of a finite duration sequences x(n) then

$$DFT\{x^*(n)\} = X^*(N-k) = X^*\big((-k)\big)_N$$

Proof

$$DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}}$$

$$DFT\{x^*(n)\} = \sum_{n=0}^{N-1} x^*(n)e^{-\frac{j2\pi kn}{N}}$$

$$DFT\{x(n)\} = \left[\sum_{n=0}^{N-1} x(n)e^{\frac{j2\pi kn}{N}}\right]^*$$

$$e^{-\frac{j2\pi nN}{N}} = e^{-j2\pi n} = 1$$

$$= \left[\sum_{n=0}^{N-1} x(n)e^{\frac{j2\pi kn}{N}}e^{-\frac{j2\pi nN}{N}}\right]^*$$

$$= \left[\sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi n(N-k)}{N}}\right]^*$$

$$DFT\{x(n)\} = [X(N-k)]^*$$

Q) Let X(k) be a 14 point DFT of a length 14 real sequence x(n). The first 8 samples of X(k) are given by,

X(0) = 12,

X(1) = -1+3j ,

X(2) = 3+4j,

X(3) = 1-5j,

X(4) = -2+2j,

X(5) = 6+3j,

X(6) = -2-3j,

X(7) = 10. Determine the remining samples

$$DFT\{x(n)\} = [X(N-k)]^*$$

Solution

Given N=14

For n=8 ➔ $X(8) = X^*(N-k) = X^*(14-8) = X^*(6) = -2+3j$

For n=9 ➔ $X(9) = X^*(N-k) = X^*(14-9) = X^*(5) = 6-3j$

For n=10 ➔ $X(10) = X^*(N-k) = X^*(14-10) = X^*(4) = -2-2j$

For n=11 ➔ $X(11) = X^*(N-k) = X^*(14-11) = X^*(3) = 1+5j$

For n=12 ➔ $X(12) = X^*(N-k) = X^*(14-12) = X^*(2) = 3-4j$

For n=13 ➔ $X(13) = X^*(N-k) = X^*(14-13) = X^*(1) = -1-3j$

Linear Convolution

Consider a discrete sequence x(n) of length L and impulse sequence h(n) of length M, the equation for linear convolution is

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Where length of y(n) is L+M-1

Let's discuss it with an example

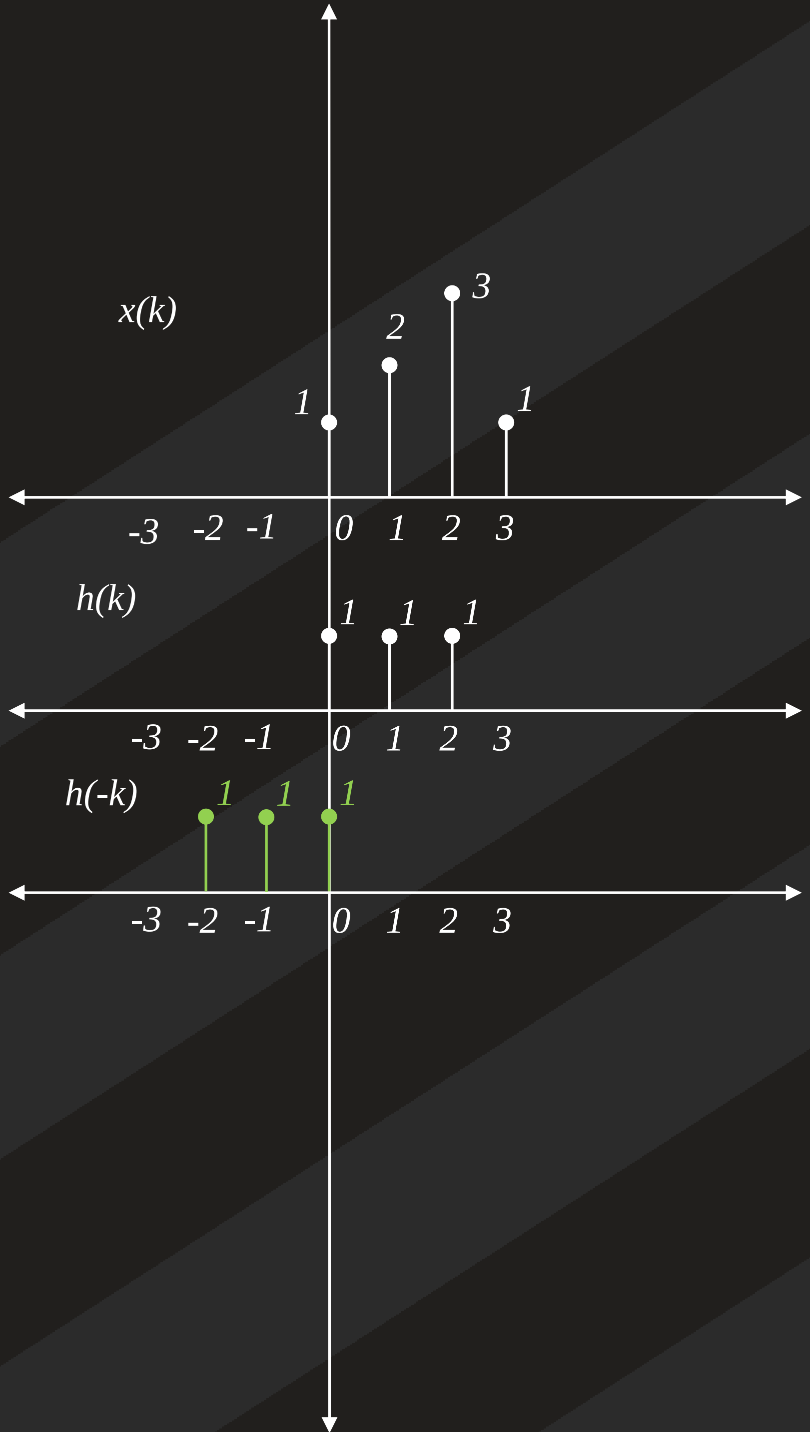
Q) Find the convolution of x(n) = {1,2,3,1}, h(n)={1,1,1,}

Solution

L = 4, M = 3

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$ *Of length → 4+3-1 = 6*

For n=0 → $$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = (1.0) + (1.0) + (1.1) + (0.2) + (0.3) + (0.1) = 1$$



For n=1 ➔ $y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = \sum_{k=-\infty}^{\infty} x(k)h(-k+1)$

$= (1.0) + (1.1) + (1.2) + (0.3) + (0.1) = 3$

For n=2 ➔ $y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = \sum_{k=-\infty}^{\infty} x(k)h(-k+2)$

$= (1.1) + (1.2) + (1.3) + (0.1) = 6$

For n=3 ➔ $y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k) = \sum_{k=-\infty}^{\infty} x(k)h(-k+3)$

$= (0.1) + (1.2) + (1.3) + (1.1) = 6$

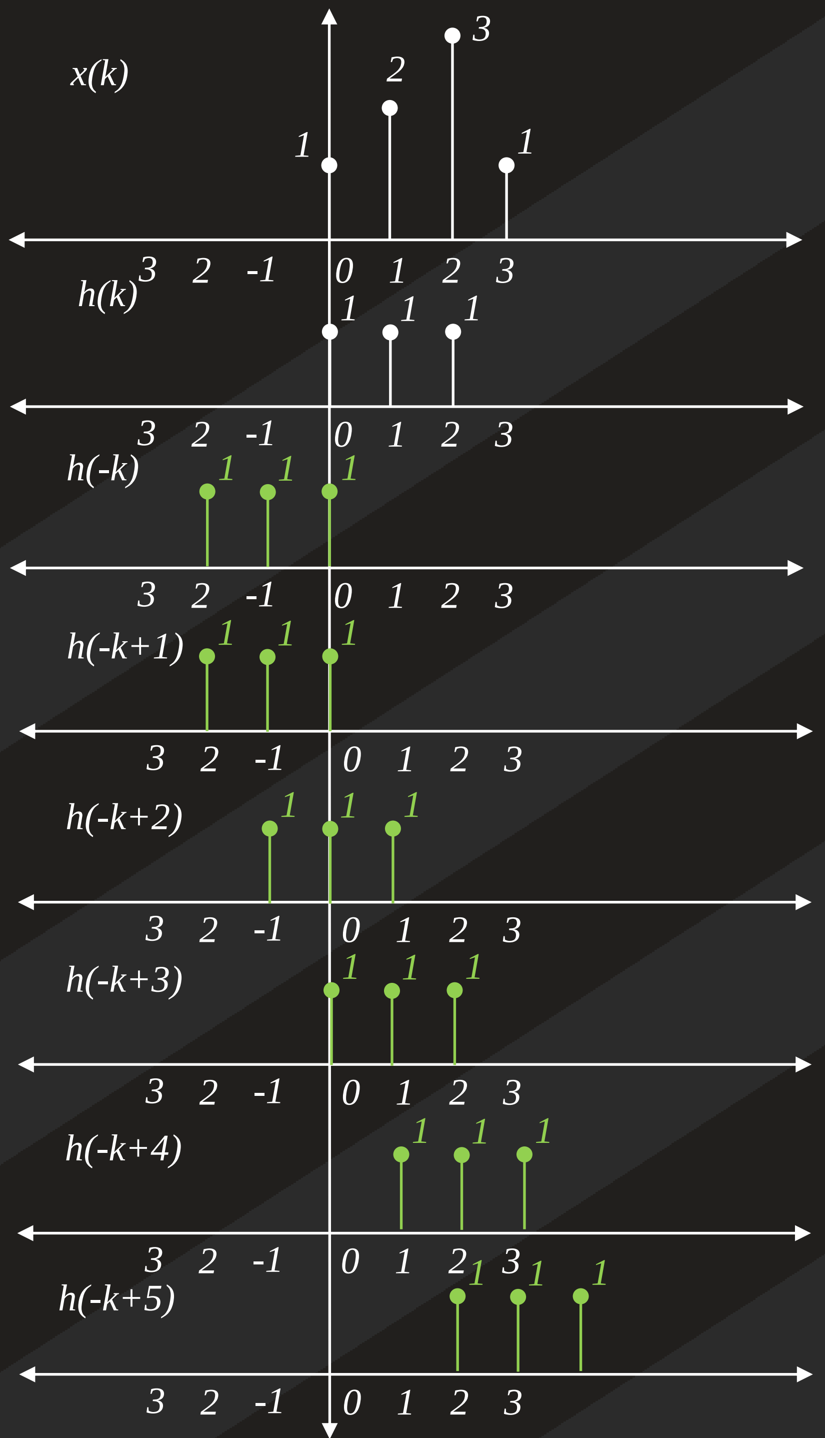
For n=4 ➔ $y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k) = \sum_{k=-\infty}^{\infty} x(k)h(-k+4)$

$= (0.1) + (0.2) + (1.3) + (1.1) = 4$

For n=5 ➔ $y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k) = \sum_{k=-\infty}^{\infty} x(k)h(-k+5)$

$= (0.1) + (0.2) + (0.3) + (1.1) = 1$

$y(n) = \{1,3,6,6,4,1\}$



Q) Find the convolution of x(n) = {1,2,3,1}, h(n)={1,1,1,}

Solution

	1	1	1
1	1	1	1
2	2	2	2
3	3	3	3
1	1	1	1

$$y(n) = \{1,3,6,6,4,1\}$$

Circular Convolution

Consider two discrete sequence $x_1(n)$ & $x_2(n)$ of length N with DFTs $X_1(k)$, $X_2(k)$

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m) x_2((n-m))_N$$

$$x_3(n) = x_1(n) \odot x_2(n)$$

Also

$$DFT\{x_1(n) \odot x_2(n)\} = X_1(k).X_2(k)$$

Matrix method

Let's discuss it with an example

Q) Find the circular convolution of x(n) = {1,2,3,4}, h(n)={1,-1,1,}

Solution

L = 4, M = 3

Since lengths are not same we do zero-padding

$$h(n) = \{1,-1,1,0\}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 3 \\ 3 & 1 & 4 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (1.1)+(4.-1)+(3.1)+(2.0) = 0 \\ (2.1)+(1.-1)+(4.1)+(3.0) = 5 \\ (3.1)+(2.-1)+(1.1)+(4.0) = 2 \\ (4.1)+(3.-1)+(2.1)+(2.0) = 3 \end{bmatrix}$$

$$y(n) = \{0,5,2,3\}$$

Circular Convolution

Concentric Circle method / Stockholm's Method

Let's discuss it with an example

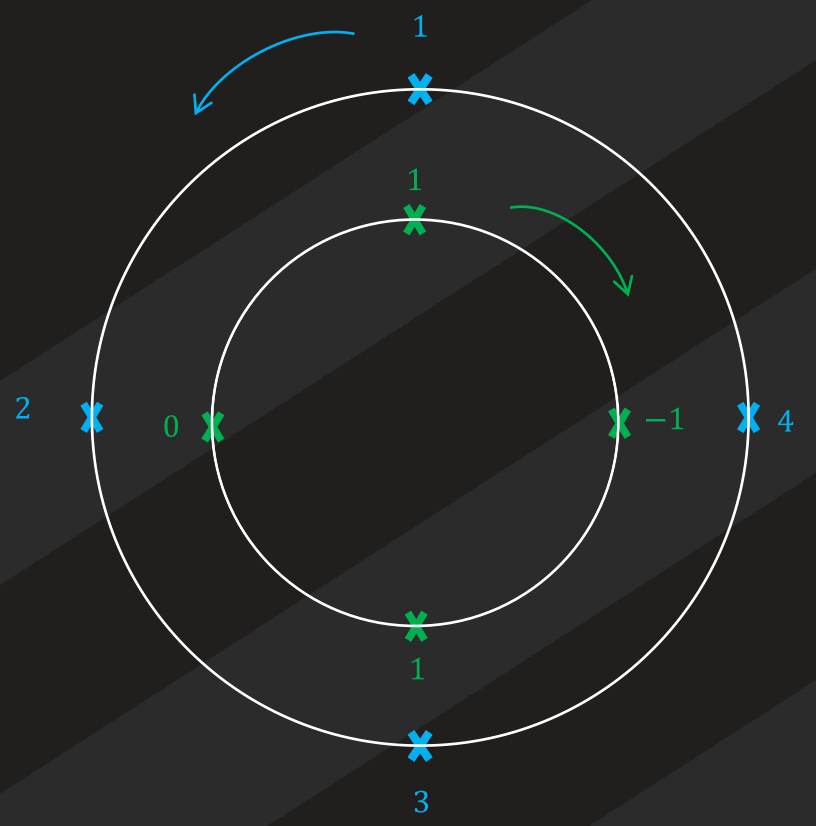
Q) Find the circular convolution of x(n) = {1,2,3,4}, h(n)={1,-1,1,}

Solution

L = 4, M = 3

Since lengths are not same we do zero-padding

$$h(n) = \{1, -1,1,0\}$$



For n=0 ➔ $y(0) = (1.1) + (2.0) + (3.1) + (4.-1) \quad = 0$

For n=1 ➔ $y(1) = (1.-1) + (2.1) + (3.0) + (4.1) \quad = 5$

For n=2 ➔ $y(2) = (1.1) + (2.-1) + (3.1) + (4.0) \quad = 2$

For n=3 ➔ $y(3) = (1.0) + (2.1) + (3.-1) + (4.1) \quad = 3$

$$y(n) = \{0,5,2,3\}$$

Linear convolution using circular convolution

Let there are two sequence x(n) with L and h(n) with length M. in linear convolution the length of output in L+M-1. In circular convolution the length of the both input is L=M

Let's discuss with an example

Q) Find the convolution of the sequences x(n) = {1,2,3,1}, h(n)={1,1,1,}

First we have to make the length of the x(n) and h(n) by adding zeros

$x(n) = \{1,2,3,1,0,0\}$ *(M-1 zeros)*

$h(n) = \{1,1,1,0,0,0\}$ *(L-1 zeros)*

$$\begin{bmatrix} 1 & 0 & 1 & 3 & 2 \\ 2 & 0 & 0 & 1 & 3 \\ 3 & 1 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (1.1)+(0.1)+(0.1)+(1.0)+(3.0)+(2.0) = 1 \\ (2.1)+(1.1)+(0.1)+(0.0)+(1.0)+(3.0) = 3 \\ (3.1)+(2.1)+(1.1)+(0.0)+(0.0)+(1.0) = 6 \\ (1.1)+(3.1)+(2.1)+(1.0)+(0.0)+(0.0) = 6 \\ (0.1)+(1.1)+(3.1)+(2.0)+(1.0)+(0.0) = 4 \\ (0.1)+(0.1)+(1.1)+(3.0)+(2.0)+(1.0) = 1 \end{bmatrix}$$

$y(n) = \{1,3,6,6,4,1\}$

Filtering of long duration sequences

1) Overlap – save method

Let's consider an input sequence x(n) of length L_s *and response h(n)of length M, the steps to follow overlap – save method is*

Step 1 : input x(n) is divided into length L (L≥M)

Step 2 : Calculate the length N=L+M-1

Step 3 : Add M-1 zeros to the start to first segment, each segment (length = L) has its first M-1 points coming from previous segment, making each of length N

Step 4 : Make impulse response to length N by adding zeros

Step 5 ; Find the circular convolution of each new segments with new h(n)

Step 6 : Linearly combine each results and take sequence of length L_s*+M-1 from that by discarding/removing first M-1 points*

1) Overlap – save method

Q) Find the convolution of the sequences x(n) = {3,-1,0,1,3,2,0,1,2,1} and h(n) ={1,1,1}

Solution

Given , Ls = 10 & M=3 *Lets guess the value of L =3* $(L \geq M)$

Step 1 : input x(n) is divided into length L

$x_1(n) = \{3, -1, 0\}$

$x_2(n) = \{1,3,2\}$

$x_3(n) = \{0,1,2\}$

$x_4(n) = \{1,0,0\}$

Step 2 : Calculate the length N=L+M-1

$N = L + M - 1 \quad = 3 + 3 - 1 = 5$

1) Overlap – save method

Step 3 : Add M-1 zeros to the start to first segment, each segment (length = L) has its first M-1 points coming from previous segment, making each of length N

$$x_1(n) = \{0,0,3,-1,0\}$$

$$x_2(n) = \{-1,0,1,3,2\}$$

$$x_3(n) = \{3,2,0,1,2\}$$

$$x_4(n) = \{1,2,1,0,0\}$$

M-1 = 3-1 = 2

$$x_1(n) = \{3,-1,0\}$$

$$x_2(n) = \{1,3,2\}$$

$$x_3(n) = \{0,1,2\}$$

$$x_4(n) = \{1,0,0\}$$

Step 4 : Make impulse response to length N by adding zeros

$$h(n) = \{1,1,1,0,0\}$$

1) Overlap – save method

Step 5 ; Find the circular convolution of each new segments with new h(n)

$$y_1(n) = x_1(n) \odot h(n) = \{0,0,3,-1,0\} \odot \{1,1,1,0,0\} = \{-1,0,3,2,2\}$$

$$y_2(n) = x_2(n) \odot h(n) = \{-1,0,1,3,2\} \odot \{1,1,1,0,0\} = \{4,1,0,4,6\}$$

$$y_3(n) = x_3(n) \odot h(n) = \{3,2,0,1,2\} \odot \{1,1,1,0,0\} = \{6,7,5,3,3\}$$

$$y_4(n) = x_4(n) \odot h(n) = \{1,2,1,0,0\} \odot \{1,1,1,0,0\} = \{1,3,4,3,1\}$$

$$\begin{bmatrix} 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 3 \\ 3 & 0 & 0 & 0 & -1 \\ -1 & 3 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Step 6 : Linearly combine each results and take sequence of length L_s+M-1 from that by discarding/removing first M-1 points

M-1 = 3-1 = 2

$$y(n) = \{3,2,2,0,4,6,5,3,3,4,3,1\}$$

Check whether length of y(n) is L_s+M-1 , if yes discard the higher sequences

L_s+M-1 = 10+3-1 = 12

1) Overlap – save method

Q) Find the convolution of the sequences x(n) = {1,2,-1,2,3,-2,-3,-1,1,1,2,-1} and h(n) ={1,2} using overlap-save method

Solution

Given , Ls = 12 & M=2 Lets guess the value of L =3 $(L \geq M)$

Step 1 : input x(n) is divided into length L

$$x_1(n) = \{1, 2, -1\}$$

$$x_2(n) = \{2,3, -2\}$$

$$x_3(n) = \{-3, -1,1\}$$

$$x_4(n) = \{1,2, -1\}$$

Step 2 : Calculate the length N=L+M-1

$$N = L + M - 1 \quad = 3 + 2 - 1 = 4$$

1) Overlap – save method

Step 3 : Add M-1 zeros to the start to first segment, each segment (length = L) has its first M-1 points coming from previous segment, making each of length N

$$x_1(n) = \{0,1,2,-1\}$$

$$x_2(n) = \{-1,2,3,-2\}$$

$$x_3(n) = \{-2,-3,-1,1\}$$

$$x_4(n) = \{1,1,2,-1\}$$

$$x_5(n) = \{-1,0,0,0\}$$

M-1 = 2-1 = 1

$$x_1(n) = \{1,2,-1\}$$

$$x_2(n) = \{2,3,-2\}$$

$$x_3(n) = \{-3,-1,1\}$$

$$x_4(n) = \{1,2,-1\}$$

Step 4 : Make impulse response to length N by adding zeros

$$h(n) = \{1,2,0,0\}$$

1) Overlap – save method

Step 5 ; Find the circular convolution of each new segments with new h(n)

$$y_1(n) = x_1(n) \odot h(n) = \{0,1,2,-1\} \odot \{1,2,0,0\} = \{-2, 1, 4, 3\}$$

$$y_2(n) = x_2(n) \odot h(n) = \{-1,2,3,-2\} \odot \{1,2,0,0\} = \{-5, 0, 7, 4\}$$

$$y_3(n) = x_3(n) \odot h(n) = \{-2,-3,-1,1\} \odot \{1,2,0,0\} = \{0, -7, -7, -1\}$$

$$y_4(n) = x_4(n) \odot h(n) = \{1,1,2,-1\} \odot \{1,2,0,0\} = \{-1, 3, 4, 3\}$$

$$y_5(n) = x_5(n) \odot h(n) = \{-1,0,0,0\} \odot \{1,2,0,0\} = \{-1, -2, 0, 0\}$$

$$\begin{bmatrix} 0 & -1 & 2 & 1 \\ 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

Step 6 : Linearly combine each results and take sequence of length L_s+M-1 from that by discarding/removing first M-1 points

$$y(n) = \{1,4,3,0,7,4,-7,-7,-1,3,4,3,-2,0,0\}$$

Check whether length of y(n) is L_s+M-1 , if yes discard the higher sequences

$L_s+M-1 = 12+2-1 = 13$

$$y(n) = \{1,4,3,0,7,4,-7,-7,-1,3,4,3,-2\}$$

Filtering of long duration sequences

2) Overlap – add method

Let's consider an input sequence x(n) of length L_1 and response h(n) of length M, the steps to follow overlap – save method is

Step 1 : input x(n) is divided into length L (L≥M)

Step 2 : Calculate the length N=L+M-1

Step 3 : Add M-1 zeros on each segment (length = L) of x(n)

Step 4 : Make impulse response to length N by adding zeros

Step 5 ; Find the circular convolution of each new segments with new h(n)

Step 6 : Add last and first M-1 points of each segments, discard/remove excess point than L_1+M-1

1) Overlap – add method

Q) Find the convolution of the sequences x(n) = {3,-1,0,1,3,2,0,1,2,1} and h(n) ={1,1,1}

Solution

Given , L_1 = 10 & M=3 Lets guess the value of L =3 $(L \leq M)$

Step 1: input x(n) is divided into length L (L≥M)

$$x_1(n) = \{3, -1, 0\}$$

$$x_2(n) = \{1,3,2\}$$

$$x_3(n) = \{0,1,2\}$$

$$x_4(n) = \{1,0,0\}$$

Step 2 : Calculate the length N=L+M-1

$$N = L + M - 1 \quad = 3 + 3 - 1 = 5$$

1) Overlap – add method

Step 3 : Add M-1 zeros on each segment (length = L) of x(n)

$x_1(n) = \{3, -1, 0, 0, 0\}$

$x_2(n) = \{1, 3, 2, 0, 0\}$

$x_3(n) = \{0, 1, 2, 0, 0\}$

$x_4(n) = \{1, 0, 0, 0, 0\}$

M-1 = 3-1 = 2

$x_1(n) = \{3, -1, 0\}$

$x_2(n) = \{1,3,2\}$

$x_3(n) = \{0,1,2\}$

$x_4(n) = \{1,0,0\}$

Step 4 : Make impulse response to length N by adding zeros

$h(n) = \{1,1,1,0,0\}$

1) Overlap – add method

Step 5 ; Find the circular convolution of each new segments with new h(n)

$$y_1(n) = x_1(n) \odot h(n) = \{3, -1, 0, 0, 0,\} \odot \{1,1,1,0,0\} = \{3,2,2,-1,0\}$$

$$y_2(n) = x_2(n) \odot h(n) = \{1, 3, 2, 0, 0\} \odot \{1,1,1,0,0\} = \{1,4,6,5,2\}$$

$$y_3(n) = x_3(n) \odot h(n) = \{0, 1, 2, 0, 0\} \odot \{1,1,1,0,0\} = \{0,1,3,3,2\}$$

$$y_4(n) = x_4(n) \odot h(n) = \{1, 0, 0, 0, 0\} \odot \{1,1,1,0,0\} = \{1,1,1,0,0\}$$

$$\begin{bmatrix} 3 & 0 & 0 & 0 & -1 \\ -1 & 3 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 \\ 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Step 6 : Add last and first M-1 points of each segments, discard/remove excess point than L_1+M-1

$\{3, 2, 2, -1, 0\}$

$\{1, 4, 6, 5, 2\}$

$\{0,1, 3, 3, 2\}$

$\{1, 1, 1, 0, 0\}$

$\{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1, 0,0\}$

Check whether length of y(n) is L_1+M-1 , if yes discard the higher sequences

L_1+M-1 = 10+3-1 = 12

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

1) Overlap – add method

Q) Find the convolution of the sequences x(n) = {1,2,-1,2,3,-2,-3,-1,1,1,2,-1} and h(n) ={1,2} using overlap-add method

Solution

Given , Ls = 12 & M=2 Lets guess the value of L =3 $(L \geq M)$

Step 1 : input x(n) is divided into length L

$x_1(n) = \{1, 2, -1\}$

$x_2(n) = \{2,3, -2\}$

$x_3(n) = \{-3, -1,1\}$

$x_4(n) = \{1,2, -1\}$

Step 2 : Calculate the length N=L+M-1

$N = L + M - 1 \quad = 3 + 2 - 1 = 4$

1) Overlap – save method

Step 3 : Add M-1 zeros on each segment (length = L) of x(n)

$x_1(n) = \{1, 2, -1\}$

$x_2(n) = \{2,3, -2\}$

$x_3(n) = \{-3, -1,1\}$

$x_4(n) = \{1,2, -1\}$

M-1 = 2-1 = 1

$x_1(n) = \{1, 2, -1, 0\}$

$x_2(n) = \{2,3, -2,0\}$

$x_3(n) = \{-3, -1,1,0\}$

$x_4(n) = \{1, 2, -1,0\}$

Step 4 : Make impulse response to length N by adding zeros

$h(n) = \{1, 2 , 0 , 0 \}$

1) Overlap – save method

Step 5 ; Find the circular convolution of each new segments with new h(n)

$$y_1(n) = x_1(n) \odot h(n) = \{1,2,-1,0\} \odot \{1,2,0,0\} = \{1,4,3,2\}$$

$$y_2(n) = x_2(n) \odot h(n) = \{2,3,-2,0\} \odot \{1,2,0,0\} = \{2,7,4,-4\}$$

$$y_3(n) = x_3(n) \odot h(n) = \{-3,-1,1,0\} \odot \{1,2,0,0\} = \{-3,-7,-1,2\}$$

$$y_4(n) = x_4(n) \odot h(n) = \{1,2,-1,0\} \odot \{1,2,0,0\} = \{1,4,3,-2\}$$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

Step 6 : Add last and first M-1 points of each segments, discard/remove excess point than L1+M-1

$$\{1, 4, 3, 2\}$$
$$\{-2, 7, 4, -4\}$$
$$\{-3, -7, -1, 2\}$$
$$\{1, 4, 3, -2\}$$

$$\{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2, \}$$

Check whether length of y(n) is L_s+M-1 , if yes discard the higher sequences

$$L_s + M - 1 = 12 + 2 - 1 = 13$$

$$y(n) = \{1,4,3,0,7,4,-7,-7,-1,3,4,3,-2\}$$